Book Review

The Hilbert Challenge

by Jeremy J. Gray, Oxford University Press, 2000, 315 pp + xii.

Reviewed by **Leong Yu Kiang**

The subject of this book is about the most well-known and most celebrated collection of landmark problems presented by one of the greatest mathematicians of the 20th Century. In 1900, at the Paris meeting of the International Congress of Mathematicians, the German mathematician David Hilbert (1862 - 1945) gave a talk which has ever since generated a tremendous influence on the development of mathematics and possibly on the lives and careers of talented mathematicians, especially ambitious young mathematicians around the world. Hilbert himself was a universalist who would enter one area of mathematics at a time for a short period and then made his exit leaving behind milestones that altered the future landscape of the area.

Hilbert was a great problem solver who introduced ideas and methods which are often controversial. The layman may be surprised to know that mathematical research can be controversial and that mathematicians do squabble over style, taste and the best way to prove theorems. One example is Hilbert's solution of a fundamental problem about the existence of a finite basis in the theory of invariants. His method was simple and yet so radical that the leading practitioners of the field dubbed it as theology rather than mathematics. Perhaps there is no other area of human knowledge that offers such a wide gamut of tantalizing problems the solution of just one of which would ensure the problem solver a permanent place in the pantheon of mathematical trail blazers.

The book by Gray is captivating in both the mathematics that is shaped by the 23 Hilbert problems and in the history of the subsequent mathematical research in these problems, especially in the post-second world war developments. Even if you do not understand the technicalities of the field, you could feel the pulse, if not excitement, of the race towards the solution of particular problems. Often, the prize of the race is the coveted Fields Medal - arguably the most prestigious honour that can be bestowed on a young mathematician (an unofficial rule of the awards committee effectively setting an age limit of 40 years). The importance of the Hilbert problems is such that careers are made from their solutions and others possibly ruined from nobody knows how many lifetimes of futile attempts.

The main areas covered by the Hilbert are set theory, logic, geometry, Lie groups, axioms of physics and probability, number theory, theory of algebraic invariants, topology, theory of functions, calculus of variations and partial differential equations,

The author also traces the biography of Hilbert in all its poignancy through the political conflicts that violently divided Europe during his lifetime. It is also striking that such conflicts did not stifle the progress that was continuously made towards the solution of important mathematical problems. However, there was a stain on the history of mathematical ideas brought about by the racial politics of the Nazis and then by the incongruous politics of the Cold War. Mathematical research can be subjective in its judgement of personal ideological inclinations, sometimes made even more rigid by the logical rigour of mathematics itself.

Book Review

An English translation of Hilbert's epoch-making lecture delivered in German is given in the book. It gives you a direct insight into a great mind and its style may even give you a hint of the social inclinations of his time. Many of the Hilbert problems have been solved and there was a meeting organised by the American Mathematical Society in 1974 to review not just their status but the rich legacy bequeathed by the efforts that paved the way towards their solutions. More recently, in 2000 (one hundred years after Hilbert's Paris lecture) the University of California at Los Angeles organised an ambitious meeting on "Mathematical Challenges of the 21st Century" in the spirit of Hilbert, except that it now requires some 36 mathematicians, computer scientists and physicists to map out the landmark problems of the mathematical sciences in the new century.

This book offers you a glimpse of the legacy of the Hilbert problems and with sufficient details to reveal the grandeur and splendour of the collective enterprise generated by the problems. It should enthral the beginning research student in mathematics and physics while anyone with a serious interest in mathematics will be intellectually enriched by the sweeping panorama of ideas and events.

Leong Yu Kiang
23 February 2004